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Innovation in Portfolio Optimization through the Use of Genetic Algorithms for Sustainable Entrepreneurship in Volatile Markets

Innovación en la Optimización de Carteras mediante el uso de Algoritmos Genéticos para Emprendimientos Sostenibles en Mercados Volátiles

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ABSTRACT

Context. In the global financial landscape, marked by heightened economic volatility and constant transformations, entrepreneurs face the challenge of identifying sustainable investment strategies to ensure effective long-term risk management.

Purpose. This study aims to develop a model for optimizing investment portfolios through financial and technological tools to maximize returns in highly volatile environments. Moreover, it aligns with the principles outlined in the Oslo Manual and the United Nations Sustainable Development Goals (**SDGs**).

Methodology. Markowitz portfolio optimization using a classic genetic algorithm applied to data from 10 companies selected from the technology, health, and finance sectors. This data, obtained from Yahoo Finance, covers the period from 2020 to 2023. The reliability of the models was rigorously validated through internal consistency analysis, ensuring their robustness.

Theoretical and Practical Findings. Theoretical results confirm the applicability of genetic algorithms in optimizing diversified portfolios. In practice, their potential to encourage investments in sustainable companies is evident, aligning with the **SDGs** by fostering key areas such as industrial innovation.

Originality. This study adopts a multidisciplinary approach by integrating finance and technology in the selection of investment portfolios. The literature review highlights how the synergy between these two fields promotes sustainable development.

Conclusions and limitations. Findings underscore the potential of genetic algorithms to perform in highly volatile contexts. However, the reliance on historical data analysis alone highlights the need for additional studies in real-world environments. These could focus on comparing other optimization models and exploring their impact in regions with diverse market structures.

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RESUMEN

Contexto. En un panorama financiero global, marcado por una mayor volatilidad económica y transformaciones constantes, los empresarios enfrentan el desafío de identificar estrategias de inversión sostenibles, para garantizar una gestión eficaz de los riesgos a largo plazo.

Problema. En un entorno económico caracterizado por una volatilidad creciente, los empresarios se enfrentan al desafío crítico de identificar inversiones sostenibles que no solo logren un equilibrio eficaz entre riesgo y rentabilidad, sino que también fomenten la resiliencia financiera a largo plazo. ¿Qué estrategias se pueden emplear para refinar la selección de carteras y evitar escenarios impredecibles?

Objetivo. Este estudio tiene como objetivo desarrollar un modelo de optimización de carteras de inversión a través de herramientas financieras y tecnológicas para maximizar los retornos en entornos altamente volátiles. Además, se alinea con los principios establecidos en el Manual de Oslo y los Objetivos de Desarrollo Sostenible (**ODS**) de las Naciones Unidas.

Metodología. Optimizacion de carteras de Markowitz utilizando algoritmos genéticos clásicos aplicados a datos de 10 empresas seleccionadas de los sectores de tecnología, salud y finanzas. Estos datos, obtenidos de Yahoo Finance, abarcan el período de 2020 a 2023. La fiabilidad de los modelos se validó rigurosamente mediante análisis de consistencia interna, lo que garantiza su robustez.

Hallazgos Teóricos y Prácticos. Este estudio adopta un enfoque multidisciplinario al integrar las finanzas y la tecnología en la selección de carteras de inversión. La revisión de la literatura destaca cómo la sinergia entre estos dos campos promueve el desarrollo sostenible.

Originalidad. La flexibilidad de los algoritmos permite adaptarse a las variaciones del mercado en diversidad de sectores, lo que resulta optimizar carteras con múltiples activos para impulsar emprendimientos sostenibles en entornos volátiles.

Conclusiones y limitaciones. Los hallazgos destacan el potencial de los algoritmos genéticos en contextos volátiles, pero señalan la limitación de basarse solo en datos históricos. Se recomienda realizar estudios en entornos reales, comparándolos con otros modelos de optimización y analizando su impacto en mercados diversos.

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1. INTRODUCTION

Investment portfolio management has become essential for entrepreneurs and aspiring entrepreneurs. This work aims to answer the questions: Which company should I invest in? How much should I invest in each one? To address these questions, we use the methodology and portfolio theory of Markowitz (Markowitz, 1959). In the search for an optimal portfolio, an optimization method called genetic algorithms is employed (Mitchell, 1998). This approach allows, from a set of previously selected companies, decisions on how much should be invested in each, thus optimizing resource allocation. This paper examines a practical case of selecting 10 companies from different sectors.

The main objective of this project is to develop a methodology to assist the entrepreneurial trend in the population, helping to decide in which companies and in what proportion to invest, using the Markowitz methodology. This methodology evaluates the quality of a portfolio based on two main criteria: expected return and associated risk. A portfolio is considered more optimal when it offers higher returns with lower risk (Markowitz, 1959). This selection aims to achieve greater diversification, thereby reducing the total portfolio risk. How can the Markowitz methodology, combined with genetic algorithms, optimize the selection of sustainable investment portfolios for entrepreneurs? This question should be addressed in the discussion and conclusion sections, where the results obtained from applying this methodology are analyzed, highlighting how it contributes to more efficient resource allocation and its impact on sustainable development.

Therefore, the hypothesis is genetic algorithms are well-suited for solving the portfolio optimization problem due to their flexibility, adaptability, and ability to navigate complex, high-dimensional asset combinations while considering both returns and risk.

2. CONTEXT DESCRIPTION

Globally, there is a growing interest in investing money not only to make it grow but also to engage in other activities without compromising personal finances. This trend reflects a shift in how investments are perceived, with a more dynamic approach replacing the traditional culture of saving. In the United States, this drive has been reinforced by prominent investors like Warren Buffet, who promote an informed investment mindset, while digital platforms and government policies encourage financial inclusion and access to key sectors such as technology and renewable

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energy. It is worth mentioning that the cryptocurrency investment fund, considered by most portfolios, is approved on the United States stock exchange. This implies that blockchain technology goes hand in hand with the financial sector, offering new investment opportunities. Therefore, multidisciplinary sector collaboration is recognized at an international level (Vega-Santana *et al*., 2024).

Mexico is not immune to this change and is home to one of the world's most prominent investors, Carlos Slim (Nava, 2024)~~,~~ Additionally, the trend toward virtual money is gaining ground in the country, driven by financial digitalization and the rise of cryptocurrencies. This not only increases accessibility to investments but also contributes to sustainable development by reducing reliance on natural resources, such as money printing. This multidimensional approach to investing seeks to foster more balanced, accessible, and sustainable economic growth both globally and within individual countries.

3. LITERATURE REVIEW

In this section we will discuss how investments are part of today's economic innovation, as well as the methodological approach that has been used until now and its improvement, including the technique of the classical genetic algorithm based on Markowitz's theory, which is one of the contributions of this work.

3.1. Entrepreneurship and innovation in portfolio optimization

Nowadays, ever-evolving entrepreneurial culture, innovation is a driving force behind economic growth. In particular, the focus on sustainable entrepreneurship within volatile markets is shaping a new cultural value that encourages social innovation. This phenomenon can be explained through the Theory of Planned Behavior (Aguilar-Cruz & Campos-Sánchez, 2024).

This work contributes to the foundational article that applies Lagrange multipliers to solve portfolio optimization (Cruz-Trejos, 2013). A crucial aspect of this research is the innovative integration of classical Markowitz theory (Markowitz, 1959) with genetic algorithms. Markowitz's theory, introduced in the 1950s, revolutionized portfolio optimization by balancing expected returns and investment risk. However, its application in practical decision-making has often been limited due to its mathematical complexity.

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What sets this approach apart is the integration of genetic algorithms, selected for their intuitive simplicity and flexibility, particularly for individuals without extensive mathematical backgrounds. This fusion of classical theory with modern computational techniques represents a fresh perspective on portfolio optimization. It addresses news challenges, making sophisticated investment strategies more accessible and adaptable to the dynamic demands of contemporary markets.

By applying the classic genetic algorithm, we hypothesize that genetic algorithms can outperform traditional optimization techniques. This combination not only provides a comprehensive solution but also an accessible approach to tackling the challenges faced by entrepreneurs and investors. This enhances the ability to optimize capital allocation, particularly in uncertain and volatile market conditions.

3.2. Contribution to SDGs

This work aligns with several **SDGs**, particularly **SDG 8 (Decent Work and Economic Growth)**, **SDG 9 (Industry, Innovation, and Infrastructure)**, and **SDG 12 (Responsible Consumption and Production)** (IMIPCADMIN, 2021). By optimizing portfolio allocation with a focus on sustainability, the proposed model helps entrepreneurs and investors make informed decisions that support long-term economic growth and innovation. This optimization ensures that investments are directed towards sectors with growth potential, while also considering the social and environmental impact of these investments.

The model fosters responsible business practices, supports sustainable industries, and encourages economic practices that align with circular economy principles. The innovation brought by genetic algorithms, which allow for real-time responsiveness to market fluctuations, ensures that portfolios remain resilient against economic shifts and contribute to a more equitable and sustainable future.

3.3. Innovation and the Oslo Manual

According to the **Oslo Manual**, innovation can be defined as the introduction of a new or significantly improved product, process, or method (PRICIT, 2024), . In this context, the innovative nature of this research lies in the application of classic genetic algorithms to the **Markowitz**

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portfolio optimization model, creating a hybrid approach that combines classic financial theory with advanced computational techniques.

This innovation brings several key advantages:

- **Flexibility and Adaptability**: Genetic algorithms are capable of adapting to complex, dynamic market conditions, providing investors with more flexibility than traditional optimization techniques.
- **Accessibility**: By simplifying the portfolio optimization process, genetic algorithms enable individuals without deep mathematical backgrounds to engage in complex financial decision-making, democratizing access to sophisticated financial tools.
- **Efficiency and Scalability**: The model is designed to be scalable and efficient, enabling its application to a wide range of companies and investment scenarios, which can be adapted to the specific needs of different investors.

By bridging classical financial theory with cutting-edge algorithms, this work introduces an innovative methodology that has the potential to transform how portfolios are optimized in real-world financial markets. In the **Table 1** is showing the process and model flow of this work. This structured approach demonstrates not only a technical contribution to financial optimization but also an innovative methodology that supports sustainable investment practices. Through classics genetic algorithms, we enhance decision-making processes, enabling more informed, flexible, and socially responsible investment strategies.

Table 1. Process and Model Flow

Step	Description
1. Sector Identification	Identify sectors with growth potential and relevance to the current and future market dynamics.
2. Company Selection	Choose representative companies from each sector to ensure a diversified portfolio, using historical data to guide selection.
3. Algorithm Application	Apply a genetic algorithm to optimize portfolio allocation based on Markowitz's theory, maximizing returns while managing risk.
4. Portfolio Evaluation	Assess the portfolio's performance using standard financial metrics such as Sharpe ratio, expected returns, and risk.
5. Continuous Adjustment	Use the dynamic nature of genetic algorithms to adapt the portfolio to changing market conditions, ensuring real-time responsiveness.

Source: Own elaboration based on Mitchell (1998).

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3.4. Maximizing Returns: A Markowitz Portfolio Perspective

Markowitz portfolios, also known as modern portfolio theory or the Markowitz model, are a key concept in investment theory developed by Harry Markowitz in the 1950s. This theory suggests how investors can construct portfolios to maximize expected return based on a given level of risk, emphasizing the importance of diversification to reduce the portfolio's total risk (Markowitz, 1959). For example, the owner of an **asset A**, whose price changes month to month, would be interested in the asset's price for the following **month. Table 2** shows hypothetical data related to this example.

Table 2. Asset A Prices and Their Respective Returns

Mes	Precio de A ($)	Rendimiento (%)
1	100	-
2	105	5
3	98	-6.67
4	110	12.24
5	115	4.55

Source: Own elaboration using Julia 1.9.3 for calculations and LaTeX 3.0 for tables creation.

Working with the asset price as a variable would be considerably more complex under the approach provided by a Markowitz model; however, another variable of interest can be associated with the asset—its return. Defined in **Equation 1.**

Equation 1: Return formula between two periods

$$\text{Rendimiento } (\%) = \left(\frac{\text{Precio final} - \text{Precio inicial}}{\text{Precio inicial}}\right) \times 100$$

The return calculated over a period indicates how much the asset's price increased or decreased relative to its previous price. Therefore, this is the random variable of interest.

If the asset price in the fifth month is 115, considering the return with these probabilities, there are three possible values for the asset price in month 6. **Table 3** shows hypothetical data of monthly prices, their return (**R**) values, and probabilities.

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Table 3. Returns of Asset A in the fifth month.

Precio quinto mes	Valor de R	Probabilidad	Precio sexto mes
115	-.5	0.2	172.5
115	0	0.6	115
115	.5	.2	57.5

Source: Own elaboration using Julia 1.9.3 for calculations and LaTeX 3.0 for tables creation.

But these are still probabilities; what would be of interest is the expected value or mean value of these returns, **E[R].** It is expected to obtain a return of 0, which makes sense since there is an equal probability of obtaining a positive or negative return with the same value. This is the case in the example based on **Table 2,** where the calculation of the expected value of **R** is presented in **Equation 2**.

Equation 2: Calculation of expected return

$$\mathrm{E}[\mathrm{R}] = -.5 \cdot 0.2 + 0 \cdot 0.6 + .5 \cdot .2 = 0$$

Normally, an investment portfolio includes multiple assets. A portfolio with n assets is assumed, each with its respective distribution of returns and weights wi. The total return of the portfolio is **Rp**, which is a random variable that inherits the randomness of the individual returns of each asset, **Ri**, taking into account its share in the portfolio (see **Equation 3**).

Equation 3. Return for a portfolio of n assets.

$$R_p = w_1 \cdot R_1 + \cdot R_2 + \ldots + w_n \cdot R_n$$

However, the expected value of the total portfolio is of interest for a portfolio. It is represented by **Equation 4,** the formula for the expectation of the sum of random variables (Ross, 2014).

Equation 4. Expected return for a portfolio of n assets

$$E(R_p) = w_1 \cdot E(R_1) + w_2 \cdot E(R_2) + \ldots + w_n \cdot E(R_n)$$

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3.5. Risk measure

Risk is based on the variability of expected returns, and from the example presented in the previous section, the goal is to reach that theoretical definition in an equation.

Suppose two portfolios with the same return, each consisting of two assets, where the distribution function of each asset (their probabilities) is theoretically known. This hypothetical example presents its data in **Table 4.**

Table 4. Different portfolios with the same return

Cartera 1				Cartera 2			
A	**Probabilidad**	B	**Probabilidad**	C	**Probabilidad**	D	**Probabilidad**
-1	0.6	-1	0.3	.3	0.1	.5	0.2
-.5	0.2	0	0.4	.35	0.8	.55	0.6
5.25	0.2	2.83	0.3	.4	0.1	.6	0.2

Source: Own elaboration using Julia 1.9.3 for calculations and LaTeX 3.0 for tables creation.

In **Equations 5 and 6**, the calculations of the returns for both portfolios are shown, demonstrating that the result is the same.

Equation 5: Expected return for portfolio 1

$$E(R_{C_1}) = w_{1_1} \cdot E(A) + w_{2_1} \cdot E(B) = (0.5)(.35) + (0.5)(.55) = 0.45$$

Equation 6: Expected return for portfolio 2

$$E(R_{C_2}) = w_{1_2} \cdot E(C) + w_{2_2} \cdot E(D) = (0.5)(.35) + (0.5)(.55) = 0.45$$

If the choice is between two portfolios based solely on return, the decision would be indifferent, as both offer the same expected return. However, it is important to analyze the distribution of the returns for each.

Each portfolio is made up of two assets, and each has three possible returns, which results in 9 total return combinations per portfolio. For example, in **Portfolio 1**, if **Asset A** has a return of -1, **Asset B** can have returns of -1, 0, or 2.83, resulting in combinations (-1,-1), (-1,0), and (-1,2.83), with total returns of 0.18, 0.24, and 0.18, respectively.

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Since each asset has three possible returns, the portfolio has 9 combinations, and the probability of each combination is calculated by multiplying the probabilities of the returns of the two assets. For a more detailed analysis, see **Table 5.**

Table 5: Different returns for each asset in two portfolios

Cartera 1						Cartera 2					
A	Proba	B	Proba	R	Proba	A	Proba	B	Proba	R	Proba
-1	0.6	-1	0.3	-1	.18	.3	0.1	.5	0.3	.4	.18
-1	0.6	0	0.4	-.5	.24	.3	0.1	.55	0.4	.425	.24
-1	0.6	2.83	0.3	.915	.18	.3	0.1	.6	0.3	.45	.18
-.5	0.2	-1	0.3	-.75	.06	.35	0.8	.5	0.3	.425	.06
-.5	0.2	0	0.4	-.25	.08	.35	0.8	.55	0.4	.45	.08
-.5	0.2	2.83	0.3	1.165	.06	.35	0.8	.6	0.3	.475	.06
5.25	0.2	-1	0.3	2.125	.06	.4	0.1	.5	0.3	.45	.06
5.25	0.2	0	0.4	2.625	.08	.4	0.1	.55	0.4	.475	.08
5.25	0.2	2.83	0.3	4.04	.06	.4	0.1	.6	0.3	.5	.06

Source: Own elaboration using Julia 1.9.3 for calculations and LaTeX 3.0 for tables creation.

In **Portfolio 1**, the possible return values span a considerable range, from a minimum of -1 to a maximum of 4.04. This wide spectrum allows for greater variability in the potential outcomes. On one hand, there is the possibility of achieving a significantly high return of 4.04, which could be beneficial in terms of profits. However, on the other hand, there is also an associated risk, as there is a probability of losing the entire invested capital, given the minimum return of -1.

In contrast, **Portfolio 2,** although it also has 9 possible return values, has a narrower range, fluctuating between 0.4 and 0.5. This indicates lower variability in the results. In the worst case, the return would be 0.4. This example shows that considering only the return of a portfolio is not sufficient to choose between portfolios. One way to measure this variability is through the variance and standard deviation of the portfolio's return. The higher the variance, the greater the uncertainty of the values it may take around the return. Conversely, the lower the variance, the smaller this range of potential returns. In Markowitz portfolios for real financial assets, it is assumed that returns follow a normal distribution (Markowitz, 1959).

Equation 7 presents the variance of the returns for each portfolio computed in Julia Programming Lenguaje (Julia Computing, 2024), **Var(RCi),** as previously mentioned. Where ***x*** represents a return, **SOPRCi** is the set of possible return values, **E[RCi]** is the expected return corresponding to **portfolio i,** and **P[RCi = x]** corresponds to the probability that the portfolio will achieve that return.

Equation 7. Variance for both portfolios.

$$Var(R_{Ci}) = \sum_{x \in S_{oPR_{Ci}}} (x - E[R_{Ci}])^2 * P[R_{Ci} = x]$$

3.6. Classic Genetic Algorithms

Classic Genetic Algorithms (**CGAs**) are adaptive heuristic search techniques inspired by the evolutionary principles of natural selection and genetics. They function both as computational models of natural systems and as algorithmic tools to solve complex problems. According to Melanie Mitchell (1998) in her work *An Introduction to Genetic Algorithms*, **CGAs** are designed to simulate processes observed in natural evolution, where a population of candidate solutions evolves towards more optimal solutions over several generations. This process is carried out through operations that mimic natural genetic mechanisms, such as selection, crossover (recombination), and mutation. **CGAs** stand out for their ability to address optimization problems and have a wide range of applications, including machine learning, scientific modeling, and artificial life, The classic genetic algorithms models used in this project were written and developed by us in the Julia programming language (Julia Computing, 2024).

4. METHODOLOGY AND MATERIALS

The methodology of the project can be summarized in two steps: company selection and algorithm design described in **Table 1**.

4.1. Company Selection

For the company selection, 5 important sectors of the economy were chosen, and from each sector, 2 relevant companies were selected based on the largest capital (see **Table 6**).

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Table 6. Different companies from each sector

Sectores			
N	**ID**	**Nombre**	**Sector**
1	JPM	JPMorgan	Financiera
2	MA	Mastercard	Financiera
3	NVS	Novartis AG	Farmaceutica
4	AZN	AstraZeneca PLC	Farmaceutica
5	NFLX	Netflix	Comunicacion
6	CHTR	Charter Communications	Comuniacion
7	AAPL	Apple Inc	Tecnologia
8	MSFT	Microsoft	Tecnologia
9	XOM	Exxon Mobil	Energias
10	SLB	Schlumberger	Energias

Source: Own elaboration based on the selected companies (Finance, 2023).

Once the 10 companies were selected, financial data was obtained from Yahoo Finance (2023), including the monthly stock prices of these companies over the past 3 years. Using this data, the monthly returns were calculated as the percentage change in the prices of each stock, the variances of these returns were determined as a measure of dispersion relative to the mean, and the risks associated with each company were assessed based on their return variances (risk is simply the standard deviation) Computed in Julia Programming Language (Julia Computing, 2024), as shown in **Table 7**.

Table 7. Returns, Variances, and Risks of Companies.

Portafolio					
N	**ID**	**Nombre**	**Rendimiento**	**Varianza**	**Riesgo**
1	JPM	JPMorgan	1.489 %	0.0095	0.0976
2	MA	Mastercard	2.085 %	0.0077	0.0879
3	NVS	Novartis AG	1.135 %	0.0046	0.0677
4	AZN	AstraZeneca PLC	1.868 %	0.0055	0.0745
5	NFLX	Netflix	2.055 %	0.0222	0.1489
6	CHTR	Charter Communications	1.028 %	0.0094	0.0969
7	AAPL	Apple Inc	3.614 %	0.0131	0.1146
8	MSFT	Microsoft	3.38 %	0.0059	0.0766
9	XOM	Exxon Mobil	2.295 %	0.0173	0.1314
10	SLB	Schlumberger	2.663 %	0.037	0.1924

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Source: Created by the author using Julia 1.9.3 for calculations and LaTeX 3.0 for table creation with collected data (Finance, 2023).

Other calculations essential for portfolio optimization include the covariance matrix between the returns of the companies and the correlation matrix among them.

4.2. Algorithm Design

To use classic genetic algorithms computed in Julia Programming Lenguage (Julia Computing, 2024) in the present investment problem, several key elements must be defined. In the context of genetic algorithms, a way to represent an individual or portfolio with different weights assigned to the assets must be established (see **Equation 8**).

Equation 8: Genetic form of an individual

$$S = (w_1, w_2, w_3, \ldots, w_N), \quad \text{donde} \quad \sum_{i=1}^{N} w_i = 1$$

In the context of classic genetic algorithms, the individual ***S*** represents the chosen investment portfolio, where each wi is the weight assigned to each company within the portfolio. For example, if there are only 2 assets (N=2), such as Amazon and Apple, with $1,000,000 invested, divided equally between both, the representation of our portfolio would be S = (0.5, 0.5). This methodology allows for the simple representation of any portfolio with a fixed amount of money and a set number of assets, where the sum of the weights always equals 1.

When evaluating the suitability of one portfolio compared to another in genetic algorithms, several factors come into play. The goal is to determine which portfolio composition provides the highest monthly return given a specific level of risk, or simply which portfolio offers the highest return regardless of the risk. In this case, a risk level r is established, and the portfolio that maximizes the return under this risk level is sought. This evaluation is typically carried out through a fitness function, shown in **Equation 9**, which rates each individual (portfolio) based on its ability to meet the objectives.

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Equation 9. Fitness function

$$f(s) = \frac{R_s}{|\sigma_s - r| + 1},$$

Equation 10: Formula for the Return of an *"Individual"*.

$$R_s = \sum_{i=1}^{N} w_i \cdot \bar{R}_i;$$

Equation 11: Variance of an *"Individual"*.

$$\sigma_s^2 = \sum_{i=1}^{N} \sum_{j=1}^{N} w_i \cdot w_j \cdot \sigma_{ij}$$

If the risk **level (r)** of the portfolio deviates from the fixed risk chosen by the investor, the fitness function decreases. This means that the fittest individuals are those with higher returns and a risk level, where the sum of the unit in division is simply a safety method to ensure that it does not divide by zero, and does not affect the fitness function.

The fitness function within genetic algorithms is the most flexible and important part, as it determines and specifies how good one portfolio is compared to another. **Equations 12** and **13** represent our initial portfolio set and a set of values associated with the portfolio values under the fitness function.

Equation 12. Initial population set of portfolios.

$$P_0 = \{S_1, S_2, S_3, \ldots, S_n\}$$

Equation 13. Set of values associated with the population under the fitness function.

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$$C_p = \{f(S_1), f(S_2), f(S_3), \ldots, f(S_n)\}$$

After defining these two aspects, the process continues with a fixed number of individuals, the initial population ***P0***. The portfolios are generated randomly, considering that the sum of their weights must equal 1 and have positive values. Subsequently, **equation 9** is applied to evaluate each of our portfolios. These initial population portfolios are not considered as options because they were randomly generated; the objective is to select the two individuals with the best scores, referred to as *"Father"* and *"Mother"* for the generation of a new individual. Therefore, to crossover two individuals, a weighting is used to determine how many genes each should contribute depending on their fitness values, in a proportional manner (Moffat, 2024). As shown in **equation 14.**

Equation 14. Crossover methodology between two individuals

$$\text{Madre} = (w_1, w_2, w_3, \ldots, w_N) \quad p = \left\lfloor N \frac{f(\text{madre})}{f(\text{madre}) + f(\text{padre})} \right\rfloor \quad \text{Padre} = (W_1, W_2, W_3, \ldots, W_N)$$

$$\text{hijo} = (w_1, w_2, \ldots, w_p, W_{p+1}, \ldots, W_N)$$

The number of genes contributed by the mother is p, which is calculated in Equation 14, and the remaining genes come from the father. If both have the same fitness value, both contribute the same number of genes. After creating the new individual, it is inserted into the population, and to maintain the same number of individuals, the one with the lowest fitness score is eliminated. In this way, the next generation of the population can be obtained. However, something very important is missing that will provide genetic diversity to the individuals. If the algorithm is run with just this, what would be obtained is something very similar to the best individual from the first generation.

However, by randomly changing the genes of some randomly chosen individuals, this will cause one of them to undergo a beneficial mutation, making it a candidate to produce a new individual, its offspring. On the other hand, if the mutation is harmful to the individual, it will be more likely to be eliminated from the population. Therefore, this will be considered a new parameter, the probability that an individual will undergo a mutation.

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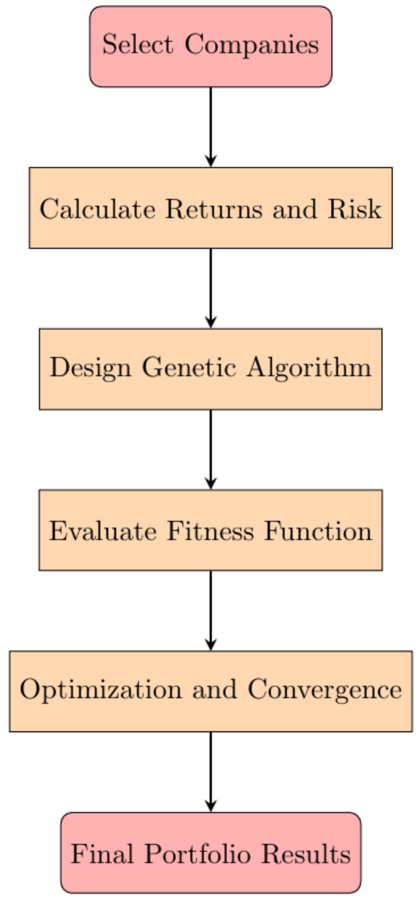
With these elements, the algorithm is ready to be implemented (see **Figure 1**). This will allow us to determine the appropriate proportion to invest in the selected companies and decide which ones to invest in.

Figure 1. Diagram of the Portfolio Optimization Process Using Genetic Algorithms and the Markowitz Model.



Source: Own elaboration using LaTeX software (version 3.0).

5. RESULTS

Before applying the classic genetic algorithm to the data, the covariance matrix (formula for covariance (Grimmet & Stirzaker, 2001) of the selected companies, shown in **Table 8**, is first presented. This matrix is based on the monthly returns calculated in **Table 7**. The diversity of results observed makes it difficult to choose which company to invest in. However, work continues with this foundation to make the optimal decision.

Table 8. Covariance Matrix between Companies

Matriz Covarianzas										
Empresa	**JPM**	**MA**	**NVS**	**AZN**	**NFLX**	**CHTR**	**AAPL**	**MSFT**	**XOM**	**SLB**
JPM	0.0087	0.0033	0.0008	0.0011	0.0006	0.0036	0.0028	0.0003	0.007	0.0128
MA	0.0033	0.0068	0.0022	0.0022	0.0002	0.0027	0.0036	0.0005	0.0057	0.0081
NVS	0.0008	0.0022	0.0043	0.0025	-0.0005	0.0013	0.0016	0.002	0.0018	0.0019
AZN	0.0011	0.0022	0.0025	0.0053	-0.0013	0.002	0.0017	0.0016	0.0019	0.0024
NFLX	0.0006	0.0002	-0.0005	-0.0013	0.022	0.0008	0.0055	0.0052	-0.0055	-0.0038
CHTR	0.0036	0.0027	0.0013	0.002	0.0008	0.008	0.004	0.0015	-0.0006	0.0028
AAPL	0.0028	0.0036	0.0016	0.0017	0.0055	0.004	0.0137	0.0053	-0.0003	0.0021
MSFT	0.0003	0.0005	0.002	0.0016	0.0052	0.0015	0.0053	0.0055	-0.0022	-0.0036
XOM	0.007	0.0057	0.0018	0.0019	-0.0055	-0.0006	-0.0003	-0.0022	0.0166	0.0207
SLB	0.0128	0.0081	0.0019	0.0024	-0.0038	0.0028	0.0021	-0.0036	0.0207	0.0331

Source: Created by the author using Julia 1.9.3 for calculations and LaTeX 3.0 for table creation, based on the monthly returns of the companies from March 2018 to March 2023.

In **Table 9**, it can be observed that there is a strong correlation between companies within the same sector, with each company showing a perfect correlation with itself. However, in general, the correlations between companies are low, which is beneficial for the selected portfolio. This low correlation means that if one company faces difficulties in the market, it will not necessarily affect all the others, helping to mitigate the risk of widespread losses. To further reduce risk, a broad diversification in the portfolio is sought, so that the good performance of some companies can offset the losses of others, providing stability and protection against adverse market movements.

Table 9. Correlation Matrix

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Matriz Correlacion										
Empresa	**JPM**	**MA**	**NVS**	**AZN**	**NFLX**	**CHTR**	**AAPL**	**MSFT**	**XOM**	**SLB**
JPM	1.0	0.4279	0.1272	0.1555	0.0415	0.4289	0.2566	0.0414	0.584	0.7521
MA	0.4279	1.0	0.4128	0.3622	0.0203	0.3687	0.3714	0.088	0.5354	0.5412
NVS	0.1272	0.4128	1.0	0.5289	-0.0547	0.2278	0.21	0.4103	0.2124	0.1618
AZN	0.1555	0.3622	0.5289	1.0	-0.1175	0.3005	0.1988	0.293	0.2061	0.1812
NFLX	0.0415	0.0203	-0.0547	-0.1175	1.0	0.0637	0.3152	0.4724	-0.2898	-0.1404
CHTR	0.4289	0.3687	0.2278	0.3005	0.0637	1.0	0.3777	0.2229	-0.0551	0.1705
AAPL	0.2566	0.3714	0.21	0.1988	0.3152	0.3777	1.0	0.6094	-0.0202	0.1002
MSFT	0.0414	0.088	0.4103	0.293	0.4724	0.2229	0.6094	1.0	-0.2274	-0.2654
XOM	0.584	0.5354	0.2124	0.2061	-0.2898	-0.0551	-0.0202	-0.2274	1.0	0.8812
SLB	0.7521	0.5412	0.1618	0.1812	-0.1404	0.1705	0.1002	-0.2654	0.8812	1.0

Source: Created by the author using Julia 1.9.3 for calculations and LaTeX 3.0 for table creation, based on the monthly returns of the companies from March 2018 to March 2023.

On the other hand, numerous experiments were conducted during the implementation of the algorithm to adjust the optimal parameters that would ensure its correct functioning. This included determining the appropriate number of generations, the optimal population size per generation, and the mutation probability. It was found that, in most cases, setting the algorithm with 10,000 generations, 50 individuals per generation, and a mutation probability of 0.01 was sufficient to achieve convergence. Although increasing the number of generations and individuals generally improves the algorithm's performance, it also significantly increases the computational cost involved in its execution.

In **Graphics 1**, it is shown that the mutation probability is a very influential factor in the algorithm's performance. A very low mutation rate causes the population quality (population quality is the sum of the fitness function of its members, see **Equation 15**) to increase very slowly over generations, while a very high mutation rate makes the population unstable.

Graphics 1. Population qualiy vs Generation

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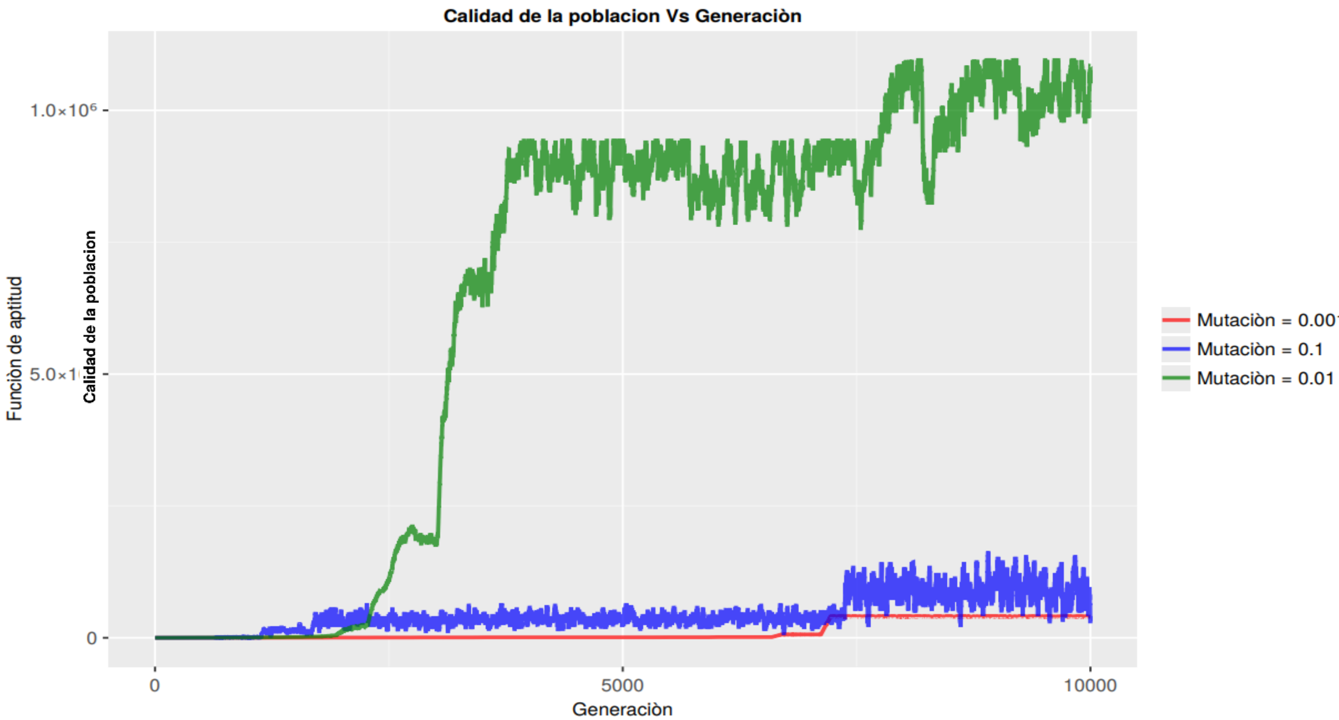

Source: Own elaboration with the help of the Julia Makie library (Moffat, 2024).

Equation 15. Population quality for a given generation.

$$calidad\,de\,poblacion = \sum_{i=1}^{n} f(s_i)$$

Experimenting with different risk levels to identify the optimal portfolio, **Table 10** is obtained. To better understand the analysis of the presented results, an example would be considering having one million pesos. The question is: how much of that money should be allocated to each company if my risk level is 0.1? The amount of money to invest in each company using the calculated weights generates a monthly return of 3.55%.

Table 10: Optimal Portfolio for Different Risk Levels.

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Pesos y rendimientos											
r	w_1	w_2	w_3	w_4	w_5	w_6	w_7	w_8	w_9	w_{10}	R_p
0.03	1.5 %	0.0 %	23.12 %	15.85 %	10.04 %	15.66 %	0.0 %	19.08 %	14.74 %	0.0 %	1.9316 %
0.05	0.0 %	5.83 %	0.0 %	15.11 %	4.05 %	8.67 %	0.0 %	47.88 %	18.46 %	0.0 %	2.6184 %
0.1	0.0 %	0.0 %	0.0 %	0.0 %	0.0 %	0.0 %	74.7 %	25.3 %	0.0 %	0.0 %	3.555 %
0.15	0.0 %	0.0 %	0.0 %	0.0 %	0.0 %	0.0 %	19.79 %	0.0 %	0.0 %	80.21 %	2.8512 %

Source: Own elaboration using Julia 1.9.3 for calculations and LaTeX 3.0 for table creation.

Therefore, the optimal portfolio when investing the specified amount of one million pesos with a risk level of 0.1 would result in the optimal portfolio being **AAPL**, Apple Inc. from the technology sector (see **Table 11**).

Table 11. Optimal Portfolio with $1,000,000 pesos and a risk level of 0.1.

Pesos									
JPM	**MA**	**NVS**	**AZN**	**NFLX**	**CHTR**	**AAPL**	**MSFT**	**XOM**	**SLB**
0	0	0	0	0	0	747,000	253,000	0	0

Source: Own elaboration using Julia 1.9.3 for calculations and LaTeX 3.0 for table creation.

6. DISCUSSION

The results of the analysis provide compelling evidence of the effectiveness of the genetic algorithm design and the thoughtful selection of companies for investment. Our findings demonstrate that the algorithm not only identifies the most suitable companies based on their risk profiles but also determines the optimal investment allocation for each company, in accordance with a specified level of risk. This approach proves to be both versatile and adaptable, offering a methodology that can be applied to diverse company sets and customized to suit individual investor preferences.

By incorporating the proposed genetic algorithm, this research contributes a novel tool to the decision-making process in investment management, allowing investors to allocate their capital more efficiently and effectively while maintaining a balance between risk and return. The results reveal that genetic algorithms significantly enhance the portfolio optimization process by exploiting their flexibility and ability to adapt dynamically to changing market conditions. This

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dynamic adaptation ensures that the algorithm remains responsive to economic shifts and the evolving needs of investors (Elton et al., 2014)

Importantly, the methodology aligns with **SDGs,** particularly **SDG 8: Decent Work and Economic Growth**, and **SDG 9: Industry, Innovation, and Infrastructure**. The genetic algorithm approach contributes to these **SDGs** by fostering more inclusive, efficient, and innovative investment strategies. Moreover, by enabling the integration of social and environmental factors into the optimization process, the algorithm promotes responsible investing practices that contribute to sustainable growth. This approach allows for a broader perspective in portfolio optimization, moving beyond traditional financial performance to include factors that support long-term sustainability.

Furthermore, the flexibility of the proposed genetic algorithm ensures that individuals who may not possess advanced mathematical knowledge can still utilize this powerful tool for portfolio optimization. By democratizing access to sophisticated financial strategies, this research makes it possible for a wider audience to engage in sustainable investing practices, thus expanding the accessibility of advanced financial decision-making processes.

6.1. Confirming Literature Theories and Contribution to the Research Question

The proposed methodology builds upon extends the existing body of literature on portfolio optimization, particularly the work of Markowitz (1959), which introduced the foundational theory of portfolio diversification and risk management. While traditional methods, such as Markowitz's mean-variance optimization, remain widely used, they often struggle with complex, non-linear relationships and multi-dimensional risk-return profiles. Evolutionary algorithms, including genetic algorithms, have been shown to provide a more adaptable solution, particularly in dynamic environments.

This study confirms the effectiveness of genetic algorithms for portfolio optimization in complex and volatile markets, offering an improvement over traditional methods.By applying classic genetic algorithms to the context of sustainable investing, this research addresses a critical gap in the literature. The findings confirm that genetic algorithms can optimize not only financial returns but also incorporate sustainability considerations, such as environmental, social, and

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governance (**ESG**) factors. This alignment with the principles of responsible investing is essential as it supports a more sustainable approach to portfolio optimization. Therefore, this work contributes to answering the central research question by demonstrating that genetic algorithms offer a scalable and flexible alternative to traditional portfolio optimization methods, capable of integrating sustainable finance objectives and responding to the growing demand for more responsible investment strategies.

6.2. Theoretical implications (*Scientia*)

This section is structured to highlight the theoretical contributions of the proposed model in sustainable development. First, the focus will be on the integration of genetic algorithms with portfolio optimization, emphasizing their innovative theoretical and implications.

6.2.1. Theoretical Contribution and Projection of the Model in Sustainable Development

This research makes a significant contribution to the theoretical framework by aligning with Markowitz's (1959) seminal work on portfolio optimization and the integration of genetic algorithms for asset allocation. By employing genetic algorithms, the study facilitates more informed decision-making for investors, optimizing the distribution of capital across various assets to maximize expected returns while mitigating risk. The results introduce new relationships between key financial variables, such as asset returns, correlations between assets, and investor risk profiles. This approach not only refines traditional methods but also opens new perspectives at the intersection of economics, mathematics, computer science, and algorithm theory, fostering the development of innovative theoretical frameworks that integrate these fields and present new methodological opportunities in financial optimization.

The innovation brought about by genetic algorithms in portfolio optimization introduces a dynamic approach, offering greater flexibility and real-time responsiveness to market fluctuations. From a sustainability perspective, the proposed methodology is set to contribute to the circular economy by promoting investments that not only aim for financial returns but also prioritize responsible practiceswithin companies. This research highlights how the use of genetic algorithms in portfolio optimization can drive innovation by providing an automated, scalable solution to the complex problem of resource allocation, thus fostering positive environmental and social impacts.

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These benefits align with the **SDGs**, particularly **SDG 8 (Decent Work and Economic Growth)**, **SDG 9 (Industry, Innovation, and Infrastructure)**, and **SDG 12 (Responsible Consumption and Production)**, which aim to promote a more inclusive and sustainable economy.

6.2.2. Current and Future Projection of the Model's Utility

The presented model not only addresses the current challenges of portfolio optimization but also holds great potential for future evolution. Currently, its utility lies in its ability to integrate a wide range of financial factors, including nonlinear risks, market fluctuations, and sustainable objectives. As the demand for responsible investment practices grows, the model's capacity to adapt to environmental, social, and governance (**ESG**) standards makes it a key tool for future financial decision-making.

In the future, this approach could become even more valuable as new machine learning techniques and data analytics evolve, allowing for greater customization and accuracy in investment portfolios. Moreover, the increasing incorporation of sustainability metrics could make this model indispensable for investors seeking to align with the **SDGs** and responsible investment principles. The flexibility of genetic algorithms to adapt to changing conditions and find optimal solutions in real-time suggests that this methodology will be crucial in addressing the forthcoming economic and environmental challenges.

This theoretical and practical advancement not only strengthens existing knowledge but also paves the way for the generation of new insights in the field of sustainable finance, providing a solid foundation for future research in portfolio optimization and responsible resource management.

6.3. Practical implications (*Praxis*)

Continuing with the practical discussion. This section is structured to highlight the future utility of the proposed model in sustainable development. The discussion will explore the current applications and the potential future evolution of the model, particularly in aligning with sustainability goals and addressing emerging challenges in financial decision-making.

6.3.1.Practical Implications and Innovation for Sustainable Development

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In practice, the use of genetic algorithms for portfolio optimization offers significant advantages for entrepreneurs and investors, especially in environments characterized by market uncertainty and the complexity of multi-asset portfolios. These complexities, which make it challenging to accurately predict returns and risks, are efficiently addressed through the flexibility and adaptability of genetic algorithms. By exploring potential asset combinations, maximizing returns while minimizing risk, these algorithms offer a robust solution to portfolio management.

This is especially valuable for entrepreneurs, providing them with powerful tools to allocate resources effectively, make informed decisions, and navigate the volatility inherent in financial markets.

Moreover, this methodology is accessible even to those who are not highly proficient in mathematics, democratizing the process of portfolio optimization. By empowering a wider range of individuals to participate in investment decision-making, it enhances financial inclusion and contributes to the broader accessibility of sophisticated investment strategies.

6.3.2. Extending the Model's Applications for Broader Impact

The potential applications of this research extend well beyond individual investment portfolios, impacting technological innovation, social groups, and the environment. Through the optimization of capital allocation, genetic algorithms support more sustainable economic practices. These practices can foster investments that promote the circular economy and encourage responsible corporate behavior, aligning with key **SDGs**, such as **SDG 8 (Decent Work and Economic Growth)**, **SDG 9 (Industry, Innovation, and Infrastructure)**, and **SDG 12 (Responsible Consumption and Production)**. The application of genetic algorithms in portfolio management thus becomes a tool not only for financial optimization but also for promoting sustainable business practices, contributing to a more equitable and environmentally conscious future.

Furthermore, this research opens the door for adapting the model to various types of investors and sectors, offering new possibilities for diverse study subjects. For example, it can be tailored to **public funds management**, ensuring that taxpayer money is allocated efficiently and responsibly. In **impact investing**, genetic algorithms can be used to optimize investments in projects with

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positive social and environmental outcomes. The model could also play a crucial role in **environmental finance**, helping investors channel funds toward initiatives that align with global sustainability targets, such as renewable energy or carbon reduction projects.

As we look to the future, the versatility of genetic algorithms in adapting to different types of investors—from individual investors to institutional ones—suggests that their application can extend to **sustainable development initiatives** across various sectors. This approach promises to improve how capital is allocated, ensuring that investments are not only financially sound but also contribute to the achievement of the SDGs, thereby helping investors contribute to positive societal and environmental change.

7. CONCLUSION

The project objective is successfully achieved, answering key questions about which companies to invest in and how much to invest to maximize returns, considering each individual's risk profile. This approach allows anyone interested in investing in specific stocks to determine how much capital to allocate to each one in an informed manner. The answers to these questions are essential for managing the income earned by entrepreneurs with the goal of investing in their innovative businesses.

7.1. How answer the question and explain the research s or hypotheses.

To address the research question of how genetic algorithms can improve the optimization of Markowitz portfolios, the hypothesis is proposed that genetic algorithms are well-suited for solving the portfolio optimization problem due to their flexibility, adaptability, and ability to navigate complex, high-dimensional asset combinations while considering both returns and risk. This study will employ simulations to compare portfolios optimized by genetic algorithms with those optimized using the classic Markowitz model (Markowitz, 1959), considering various market scenarios. The evaluation of returns, volatility, and the Sharpe ratio will help determine whether genetic algorithms provide a significant advantage in investment decision-making, particularly in uncertain and dynamic markets.

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7.2. Research findings.

The research findings demonstrate that the application of genetic algorithms for optimizing Markowitz (1959) portfolios led to efficient asset allocation across various simulated scenarios. The genetic algorithms successfully identified solutions that maximize expected returns while minimizing risk, resulting in portfolios with an optimal balance between these two key factors. Additionally, the adaptability of the algorithms to market fluctuations enhanced their ability to optimize portfolios with diverse asset combinations. These results confirm the hypothesis that genetic algorithms are a valuable tool for portfolio optimization, particularly in uncertain market conditions.

This work directly contributes to the achievement of the United Nations **SDGs**, particularly **SDG 8** (Decent Work and Economic Growth) and **SDG 9** (Industry, Innovation, and Infrastructure). By promoting efficient and sustainable financial practices, the study supports economic growth through optimized investment strategies, ultimately fostering financial stability and resilience in volatile markets. Moreover, it contributes to innovation in financial technologies (FinTech), aligning with **SDG 9** by introducing novel methodologies to improve decision-making in the investment sector.

The innovation in this research follows the principles outlined in the Oslo Manual, as it introduces an advanced application of genetic algorithms to portfolio optimization—combining classical finance theory with cutting-edge computational techniques. This interdisciplinary approach enhances the decision-making process by making sophisticated optimization models more accessible and adaptable, especially in complex and uncertain financial environments. Through this novel approach, the research paves the way for further innovation in the field of financial technology and offers practical solutions for both individual investors and institutional stakeholders.

7.3. Research final scope

The final scope of this research focuses on demonstrating the feasibility of using genetic algorithms for the optimization of Markowitz portfolios, especially in markets with uncertainty and high volatility. The results show that these algorithms can generate efficient portfolios, adapting to different investment scenarios without the need for exact knowledge of market parameters.

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However, the research was limited to the application of these algorithms in the context of diversified portfolios and did not compare them to other optimization methods. As a future extension, it would be valuable to explore the integration of genetic algorithms with other optimization techniques and conduct tests in real markets to assess their performance in more complex and dynamic conditions.

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